where

$$x = \frac{\pi\rho}{\rho_1} \quad \text{and} \quad \alpha = \log_e \frac{\alpha_2}{\alpha_1}$$

$n = 2$:

$$z = \frac{2a_1^2\beta_0\rho_1}{\pi(k_2^2 - k_1^2)}\left\{e^{qy}\left[\frac{1}{q} - \left(\frac{8q + q^3}{8q^2 + 8}\right)(q \sin y - \cos y)\right.\right.$$

$$+ \frac{12q + q^3}{48} - \left(\frac{12q^2 + q^4}{q^2 + 4}\right)(q \cos 2y + 2 \sin 2y)$$

$$\left. - \frac{q^4(2 \sin 2y + q \cos 2y + 4 \sin 4y + q \cos 4y)}{192(q^2 + 16)}\right]$$

$$- \left[\frac{1}{q} + \frac{8q + q^3}{8(q^2 + 1)} + \frac{12q + q^3}{48} - \frac{12q^3 + q^5}{q^2 + 4}\right.$$

$$\left.\left. - \frac{q^5}{96}\right]\right\} - \frac{\rho(k_2^2 - k_1^2)}{\beta_0},$$

where

$$y = 2\pi\frac{\rho}{\rho_1} \quad \text{and} \quad q = \frac{1}{\pi}\log_e\frac{a_2}{a_1}.$$

$n = 3$:

$$z = \frac{4\beta_0 a_1 a_2 \rho_1}{\pi(k_2^2 - k_1^2)}\left[x - \frac{3}{2}\alpha \sin x + \frac{9}{16}\alpha^2\left(\frac{x}{3} + \frac{\sin 2x}{2}\right)\right.$$

$$+ \left(\frac{1}{2}\alpha - \frac{27}{112}\alpha^3 + \frac{81}{1120}\alpha^5\right)\left(\sin x - \frac{\sin^3 x}{3}\right)$$

$$- \left(\frac{31}{48}\alpha^2 - \frac{99}{256}\alpha^4 - \frac{81}{6144}\alpha^6\right)\left(\frac{3x}{8} + \frac{3 \sin 2x}{16}\right.$$

$$\left. + \frac{\cos^3 x \sin x}{4}\right) + \left(\frac{\alpha^2}{8} - \frac{81}{512}\alpha^4 + \frac{81}{5120}\alpha^6\right)$$

$$\cdot \frac{\cos^5 x \sin x}{6} + \frac{9\alpha^4}{64}\frac{\sin x \cos^7 x}{8} + \left(\frac{45}{112}\alpha^3 - \frac{243}{8960}\alpha^5\right)$$

$$\left. \cdot \frac{\cos^4 x \sin x}{5} + \left(\frac{27}{256}\alpha^5 - \frac{3}{16}\alpha^3\right)\frac{\sin x \cos 6x}{7}\right] - \frac{\rho(k_2^2 - k_1^2)}{\beta_0}.$$

Acknowledgment

The author is indebted to Miss E. G. Cheatham for her assistance in carrying out most of the computations and to Mrs. M. O. Czarnomski for carrying out the initial phase of computation.

Correspondence

A Broad-Band Coaxial Ferrite Switch*

A broad-band strip-line reflective ferrite switch has been described by Johnson and Wiltse,[1] who also referred to the possibility of a similar switch in coaxial line. This note describes a coaxial on-off switch which will operate over the band 2500–4100 Mc; two such units can be combined to make a two-way switch. An isolation of 40 db was achieved, with a very low loss for the transmitting path, which, in both these devices, was obtained by magnetizing the ferrite well beyond the value for isolation.[2]

Each switching element employed a small slug of a developmental ferrite (B_{sat}=2280 gauss) which completely filled a half-inch section of air spaced coaxial line of 9/32 inch outer diameter and ⅛ inch inner diameter, the ends of which were directly coupled to Type "C" coaxial connectors. Fig. 1 shows the attenuation obtained with a field of about 400 oe compared with pads of approximately 30 db and 40 db, while Fig. 2 shows the attenuation obtained with a field of about 2500 oe compared with a 3-db pad. The attenuation of about 40 db was obtained at 100°C in a convection cooled solenoid, but greater attenuation was achieved at lower temperatures. The VSWR under reflecting conditions in Fig. 3 is about 0.15, but a lower value may be obtained over a smaller bandwidth. The solenoid power required was rather high, but was reduced by constructing the coaxial line of iron with a brass section in the position of the ferrite slug. Permanent magnet bias can also be used without unduly slowing the switching speed.

Fig. 4 shows the variation in the position of the effective short-circuit planes in front of a slug of a similar ferrite material. A two-way switch may be constructed therefore by arranging for a high impedance to appear at the T junction. Two ferrite slugs were used in the top of the "T," one of which was magnetized for isolation and the other for transmission at any instant. The transmission loss for the two-way switch is shown in Fig. 5.

The development of this switch was part of the work done under a contract for the Admiralty.

* Received by the PGMTT, June 5, 1961.

[1] C. M. Johnson and J. C. Wiltse, "A broad-band ferrite reflective switch," IRE Trans. on Microwave Theory and Techniques, vol. MTT-8, pp. 466–467; July, 1960.

[2] The General Electric Company, Ltd., Brit. Patent Application No. 19948/59; June 10, 1959.

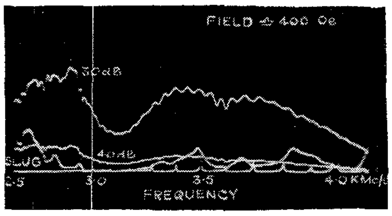

Fig. 1—Isolation of ferrite slug.

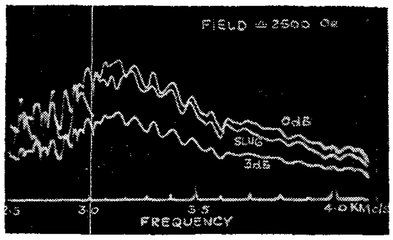

Fig. 2—Transmission loss of ferrite slug.

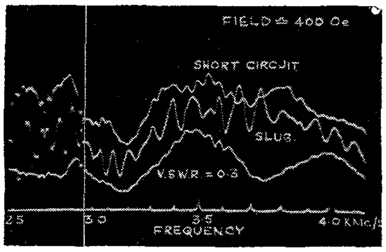

Fig. 3—Reflection coefficient of isolating slug.

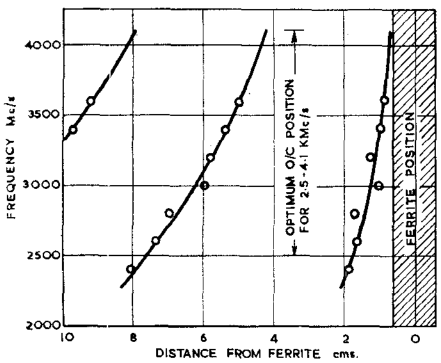

Fig. 4—Short-circuit planes produced under constant reflecting conditions.

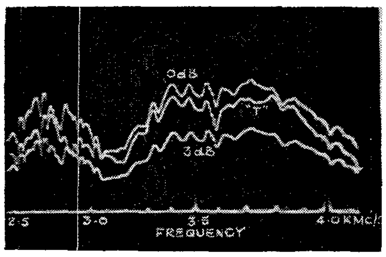

Fig. 5—Transmission loss of T junction.

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Conditions for Maximum Power Transfer*

It is sometimes of interest to ask for the conditions of maximum power transfer from a fixed source into a load constrained to vary over an arbitrary contour in the impedance plane. There exists a simple graphical solution to this question as shown below. Consider the circuit shown in Fig. 1.

If P is the power delivered to the load and $P_0 = (E_s^2/4R_s)$ is the available power from the source, then

$$\frac{P}{P_0} = \frac{4r}{(r+1)^2 + (x+x_s)^2}, \tag{1}$$

giving

$$\left[r - \left(2\frac{P_0}{P} - 1\right)\right]^2 + (x+x_s)^2 = \left(2\frac{P_0}{P} - 1\right)^2 - 1, \tag{2}$$

where

$$r = \frac{R}{R_s}, \quad x = \frac{X}{R_s}, \quad x_s = \frac{X_s}{R_s},$$

$$z = \frac{Z}{R_s}, \quad z_s = \frac{Z_s}{R_s}.$$

Eq. (2) represents a family of circles in the z plane of radius $\sqrt{(2P_0/P-1)^2-1}$, whose centers lie along the line $x=-x_s$ through the point z_s^* as shown in Fig. 2.

If a is the distance from unity to the center of a circle along the line $x=-x_s$, the equation for the family of circles becomes

$$[r - (a+1)]^2 + (x+x_s)^2 = (a+1)^2 - 1,$$

where $a = 2(P_0/P - 1)$.

To find the impedance for maximum power transfer to Z, on the plane in which z is drawn, strike a line parallel to the r axis through the point $z_s^* = 1 - jx_s$. Move an arbitrary distance a from unity along this line, and from this point draw a circle of radius

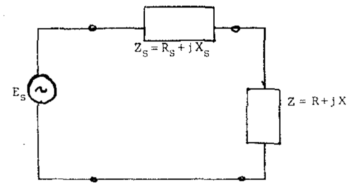

Fig. 1.

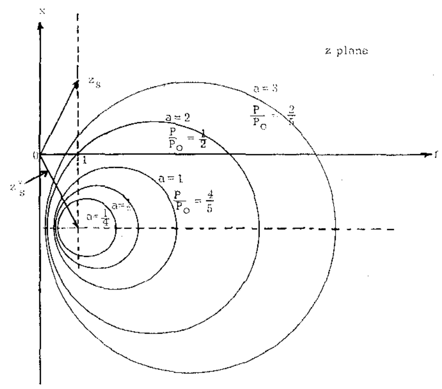

Fig. 2.

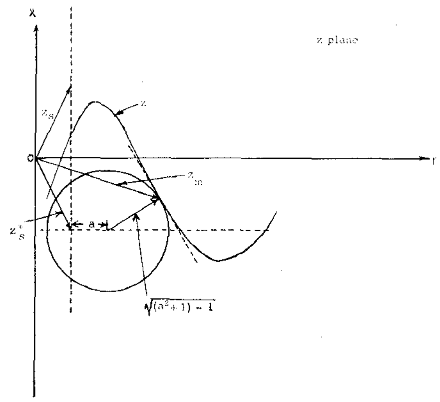

Fig. 3.

* Received by the PGMTT, June 15, 1961.